

Flow Effects on Jet Energy Loss with Detailed Balance

Luan Cheng^{a,b,c}, Jia Liu^{b,c,d} and Enke Wang^{b,c}

^a*Institute of Theoretical Physics, Dalian University of Technology, Dalian 116024, China*

^b*Institute of Particle Physics, Central China Normal University, Wuhan 430079, China*

^c*Key Laboratory of Quark & Lepton Physics (Central China Normal University), Ministry of Education, China*

^d*Department of Physics, The Ohio State University, Columbus, OH 43214, USA*

Abstract

In the presence of collective flow a new model potential describing the interaction of the hard jet with scattering centers is derived based on the static color-screened Yukawa potential. The flow effect on jet quenching with detailed balance is investigated in pQCD. It turns out, considering the collective flow with velocity v_z along the jet direction, the collective flow decreases the LPM destructive interference comparing to that in the static medium. The gluon absorption plays a more important role in the moving medium. The collective flow increases the energy gain from gluon absorption, however, decreases the energy loss from gluon radiation, which is $(1 - v_z)$ times as that in the static medium to the first order of opacity. In the presence of collective flow, the second order in opacity correction is relatively small compared to the first order. So that the total effective energy loss is decreased. The flow dependence of the energy loss will affect the suppression of high p_T hadron spectrum and anisotropy parameter v_2 in high-energy heavy-ion collisions.

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I. INTRODUCTION

One of the most striking features of nucleus-nucleus collisions at the Relativistic Heavy Ion Collider (RHIC) is the collective flow. In recent years this phenomenon has been a subject of intensive theoretical and experimental studies[1–4]. It is believed that the medium produced in nucleus-nucleus collisions at RHIC equilibrates efficiently and builds up a flow field.

Gluon radiation induced by multiple scattering of an energetic parton propagating in a dense medium leads to induced parton energy loss or jet quenching. As discovered in high-energy heavy-ion collisions, jet quenching is manifested in both the suppression of single inclusive hadron spectrum at high transverse momentum p_T region[5] and the disappearance of the typical back-to-back jet structure in dihadron correlations[6]. Extensive theoretical investigation of jet quenching has been widely carried out in recent years[7–12]. Most of the jet quenching theory research are studied in a static medium based on the static color-screened Yukawa potential proposed by Gyulassy and Wang[7]. However, the medium is not static, the collective flow need to be considered[13–15]. Later, the interaction between the jet and the target partons in the presence of collective flow was modeled by a momentum shift \mathbf{q}_0 perpendicular to the jet direction in the Gyulassy-Wang’s static potential[16], but this assumption lacks sufficient theoretical evidence. Afterwards, local transport coefficient \hat{q} , which is related to the squared average transverse momentum transfer from the medium to the hard parton per unit length, has been investigated in the presence of transverse flow[17, 18]. By applying a Lorentz boost of \hat{q} , a macroscopic result for parton energy loss based on BDMPS calculation[8] was investigated with assuming the hard parton travels with light velocity. However, this method is adaptable only for light quark energy loss. Heavy quark energy loss can not be studied in this way since its velocity is less than c . So a method to study both light and heavy quark energy loss in the presence of collective flow is needed. pQCD, a theory describing the strong interactions between quarks and gluons, can be a good selection here. Moreover, the most interesting feature of the result of jet quenching is the quadratical distance dependence of the total energy loss because of the non-Abelian nature of QCD radiation and the Landau-Pomeranchuk-Migdal (LPM) interference. Both the non-Abelian feature and LPM effect were studied in pQCD[7, 8, 10]. So that it is an interesting issue to study jet quenching in the presence of collective flow in pQCD. In addition, only radiative energy loss can be studied in Ref.[17, 18], the detailed balance effect with gluon absorption, which need to be studied in pQCD, cannot be included. It has been shown that the gluon absorption plays an important role for the intermediate jet energy region[19].

In this letter, we report a first study of the parton energy loss with detailed balance in the presence of collective flow in pQCD. We first determine the model potential to describe the interaction between the energetic jet and the scattering target partons of the moving quark-gluon medium using Lorentz boosts. Based on this new potential, we then consider both the radiation and absorption induced by the self-quenching and multiple scattering to the first order in opacity in the presence of flow. We obtains to the zeroth order opacity, the energy loss is dominated by the final-state thermal absorption. This thermal absorption is $(1 + v_z)^2$ times as that in the static medium because of different gluon distribution in the moving medium. To the first order in opacity the collective flow changes the LPM destructive interference. It reduces the jet energy loss induced by rescattering with stimulated emission depending on the direction of the flow in the positive jet direction, but hardly changes thermal absorption by rescattering. In the framework of opacity expansion developed by

Gyulassy, Lévai and Vitev (GLV)[10] and Wiedemann[11], it was shown that the higher order corrections contribute little to the radiative energy loss in the static medium. Here in the moving medium with collective flow, we will calculate the contribution from the second order in opacity to the energy loss. We are led to the conclusion that in the presence of flow, the second order in opacity correction also contributes little as in the static case. Overall, the collective flow increases energy gain, but decreases emitted energy loss, so that the total effective energy loss is decreased.

II. THE POTENTIAL MODEL

To calculate the induced radiation energy loss of jet in a static medium, the interaction potential is assumed in the Gyulassy-Wang's static model [7] that the quark-gluon medium can be modeled by N well-separated color screened Yukawa potentials,

$$V_i^a(q_i) = 2\pi\delta(q_i^0)\frac{4\pi\alpha_s}{\mathbf{q}^2 + \mu^2}e^{-i\mathbf{q}\cdot\mathbf{x}_i}T_{a_i}(j)T_{a_i}(i), \quad (1)$$

where μ is the Debye screening mass, $\alpha_s = g^2/4\pi$ is strong coupling constant, $T_{a_i}(j)$ and $T_{a_i}(i)$ are the color matrices for the jet and target parton at \mathbf{x}_i respectively. In this potential, each scattering has no energy transfer ($q_i^0 = 0$) but only a small momentum \mathbf{q} transfer with the medium. If using the four-vector potential, the Gyulassy-Wang's static potential can be denoted as $A^\mu = (V_i^a(q_i), \mathbf{A}(q_i) = 0)$.

As is well known in Electrodynamics, the static charge produces a static Coulomb electric field, while a moving charge produces both electric and magnetic field. In analogy a moving target parton in the quark-gluon medium will produce color-electric and color-magnetic fields simultaneously due to the collective flow. Therefore, the static potential model should be reconsidered.

The purpose of this paper is to study the different energy loss of the same jet in a static medium and the quark-gluon medium with collective flow. The moving medium with a velocity \mathbf{v} can be obtained through Lorentz boost of the rest frame as illustrated in Fig.1. However, since we consider energy loss of the same jet, the jet momentum need not been Lorentz transformed. So we first take a Lorentz transformation for exchanged four-momentum q , and then for four-vector potential A^μ , we can then write $A^\mu = (V_{i(flow)}^a(q_i), \mathbf{A}_{(flow)}(q_i))$ in the observer's system frame Σ' as

$$\begin{cases} V_{i(flow)}^a(q_i) = 2\pi\delta(q_i^0 - \mathbf{v} \cdot \mathbf{q})e^{-i\mathbf{q}\cdot\mathbf{x}_i}\tilde{v}(\mathbf{q})T_{a_i}(j)T_{a_i}(i), \\ \mathbf{A}_{(flow)}(q_i) = 2\pi\delta(q_i^0 - \mathbf{v} \cdot \mathbf{q})\mathbf{v}e^{-i\mathbf{q}\cdot\mathbf{x}_i}\tilde{v}(\mathbf{q})T_{a_i}(j)T_{a_i}(i), \end{cases} \quad (2)$$

where $\tilde{v}(\mathbf{q}) = 4\pi\alpha_s/(\mathbf{q}^2 - (\mathbf{v} \cdot \mathbf{q})^2 + \mu^2)$. The new potential differs from Gyulassy-Wang's static potential in that the collective flow of the quark-gluon medium produces a color-magnetic field and the flow leading to non-zero energy transfer $q_i^0 = \mathbf{v} \cdot \mathbf{q}$, which will affect jet energy loss as we will show below.

Elastic cross section for small transverse momentum transfer between jet and target partons can be deduced as

$$\frac{d\sigma_{el}}{d^2\mathbf{q}} = \frac{C_R C_2}{d_A} \frac{|\tilde{v}(\mathbf{q})|^2}{(2\pi)^2}, \quad (3)$$

where C_R and C_2 are the Casimir of jet and target parton in fundamental representation in d_R dimension, respectively. d_A is the dimension of corresponding adjoint representation.

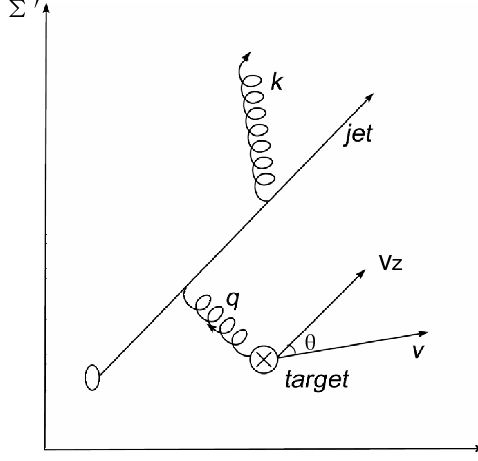


FIG. 1: View of kinematics in the quark-gluon medium with collective flow.

Our result agrees with the GLV elastic cross section in static potential when the flow velocity goes to zero [7].

III. FLOW EFFECT ON EFFECTIVE ENERGY LOSS WITH ABSORPTION

A. Energy Loss to Zeroth Order in Opacity

At zeroth order in opacity, the jet has no interaction with the target parton, we obtain the same factorized radiation amplitude off a quark

$$R^{(0)} = 2igT_c \frac{\mathbf{k}_\perp \cdot \epsilon_\perp}{\mathbf{k}_\perp^2}, \quad (4)$$

as that in the static medium in Ref. [19], where $k = [2\omega, k_\perp^2/2\omega, \mathbf{k}_\perp]$ is the four momentum of the radiated gluon with polarization $\epsilon(k) = [0, \epsilon_\perp \cdot \mathbf{k}_\perp/\omega, \epsilon_\perp]$, T_c is the color matrix, and $\alpha_s = g^2/4\pi$ is the strong coupling constant.

Taking account of stimulated emission and thermal absorption in a thermal medium with temperature T , subtracting the gluon radiation spectrum in the vacuum, one obtains the energy loss to the zeroth order in opacity,

$$\begin{aligned} \Delta E_{abs}^{(0)} &= \frac{\alpha_s C_R}{2\pi} E \int dx \int \frac{d\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2} [-P(-x)N_g(xE) + P(x)N_g(xE)\theta(1-x)] \\ &\approx -(1+v_z)^2 \frac{\pi\alpha_s C_R T^2}{3} \frac{1}{E} \left[\ln \frac{4ET}{\mu^2} + 2 - \gamma_E + \frac{6\zeta'(2)}{\pi^2} \right], \end{aligned} \quad (5)$$

where $x = \omega/E$, E is the initial jet energy. Here $N_g(|\mathbf{k}|) = 1/[\exp(k \cdot u/T) - 1] \approx 1/[\exp((1 - v_z) \cdot |\mathbf{k}|/T) - 1]$ is the thermal gluon distribution, u is the four velocity of the collective flow, v_z is the flow velocity along jet direction, the splitting function $P_{gq}(x) \equiv P(x)/x = [1 + (1-x)^2]/x$ for $q \rightarrow qg$. In Eq.(5), $\gamma_E \approx 0.5772$, $\zeta'(2) \approx -0.9376$ and T is the thermal finite temperature. Although the radiation amplitude is the same as that in the static case, the energy gain changes to $(1+v_z)^2$ times as that in the static medium because the collective flow changes the thermal gluon distribution function.

B. Energy Loss to the First Order in Opacity

When a hard parton goes through the quark-gluon medium, it will suffer multiple scattering with the parton target inside the medium. Consider the hard parton produced at $\tilde{x}_0 = (z_0, \mathbf{x}_\perp)$ with initial energy E . The hard parton interacts with the target parton at $\tilde{x}_1 = (z_1, \mathbf{x}_{1\perp})$ with flow velocity \mathbf{v} by exchanging gluon with four-momentum q , and emerges with final four-momentum p . In the light-cone components,

$$p = [E^+ + 2\mathbf{v} \cdot \mathbf{q}, \mathbf{p}_-, \mathbf{p}_\perp^+], \quad (6)$$

where $E^+ = 2E \gg \mu$.

At first order in opacity, consider the jet has the simplest case of elastic interactions with an array of the new potentials Eq.(2) localized at $\tilde{x}_i = (z_i, \mathbf{b}_i)$ using,

$$H_I(t) = \int d^3\vec{\mathbf{x}} \sum_{i=1}^N A(\vec{\mathbf{x}} - \vec{\mathbf{x}}_i) T_a(i) \phi^\dagger(\vec{\mathbf{x}}, t) T_a(R) \hat{D}(t) \phi(\vec{\mathbf{x}}, t), \quad (7)$$

where $A(\vec{\mathbf{x}} - \vec{\mathbf{x}}_i)$ is the potential we modeled with considering flow, $\hat{D}(t) = i \overleftrightarrow{\partial}_t$ and $Tr T_a(i) T_b(j) = \delta_{ij} \delta_{ab} C_2(T) d_T / d_A$.

So the scattering amplitude with one of the target partons, as compared to the static case, is changed to

$$\begin{aligned} M^{(1)} &\propto -i(2p - q)_\mu A^\mu(q) \\ &\propto 2\pi \delta(q_i^0 - \mathbf{v} \cdot \mathbf{q}) e^{-i\mathbf{q} \cdot \mathbf{x}_i} T_{a_i}(j) T_{a_i}(i) (2E + \mathbf{v} \cdot \mathbf{q}) R^{(1)}. \end{aligned} \quad (8)$$

With considering flow, the four-vector potential $A^\mu(q)$ replaces the static potential $v(q)$ [7] in the static case. One then obtains that the factorized amplitude $R^{(1)} = (1 - v_z) \tilde{v}(\mathbf{q})$ is changed by the collective flow with a factor $(1 - v_z)$.

Here we investigate the rescattering-induced radiation to the first order in opacity with considering the flow effect resulting from the moving parton target. Based on our new potential in Eq.(2) by considering collective flow of the quark-gluon medium, we obtain the factorized radiation amplitude associated with a single rescattering,

$$\begin{aligned} R^{(S1)} &= 2ig \left(\mathbf{H} T_a T_c + \mathbf{B}_1 e^{\frac{i\omega_0 z_{10}}{1-v_z}} [T_c, T_a] - 2v_z T_a T_c \mathbf{H} (1 - e^{\frac{i\omega_0 z_{10}}{1-v_z}}) + \mathbf{C}_1 e^{\frac{i(\omega_0 - \omega_1) z_{10}}{1-v_z}} [T_c, T_a] \right) \cdot \epsilon_\perp \\ &\quad \times (1 - v_z), \end{aligned} \quad (9)$$

where $z_{10} = z_1 - z_0$,

$$\omega_0 = \frac{\mathbf{k}_\perp^2}{2\omega}, \quad \omega_1 = \frac{(\mathbf{k}_\perp - \mathbf{q}_\perp)^2}{2\omega}, \quad (10)$$

$$\mathbf{H} = \frac{\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2}, \quad \mathbf{C}_1 = \frac{\mathbf{k}_\perp - \mathbf{q}_\perp}{(\mathbf{k}_\perp - \mathbf{q}_\perp)^2}, \quad \mathbf{B}_1 = \mathbf{H} - \mathbf{C}_1. \quad (11)$$

Different from the static medium case, the single rescattering amplitude depends on the collective flow of the quark-gluon medium.

The interference between the process of double scattering and no rescattering should also be taken into account to the first order in opacity [10]. Assuming no color correlation between different targets, the double rescattering corresponds to the “contact limit” of double Born

scattering with the same target [10]. Assuming the flow velocity $|\mathbf{v}| \ll 1$, with our new potential in Eq.(2) the radiation amplitude can be expressed as

$$R^{(D_1)} = 2igT_c e^{\frac{i\omega_0 z_{10}}{1-v_z}} \left(-\frac{C_R + C_A}{2} \mathbf{H} e^{-\frac{i\omega_0 z_{10}}{1-v_z}} + \frac{C_A}{2} \mathbf{B}_1 + 2v_z \frac{C_R - C_A}{2} \mathbf{H} (1 - e^{-\frac{i\omega_0 z_{10}}{1-v_z}}) \right. \\ \left. + \frac{C_A}{2} \mathbf{C}_1 e^{-\frac{i\omega_1 z_{10}}{1-v_z}} \right) \cdot \epsilon_\perp (1 - v_z)^2, \quad (12)$$

where C_A is the Casimir of the target parton in adjoint representation in d_A dimension. The double rescattering amplitude also depends on the collective flow.

To the first order in opacity, we then derive the induced radiation probability including both the stimulated emission and thermal absorption as

$$\frac{dP^{(1)}}{d\omega} = \frac{C_2}{8\pi d_A d_R} \frac{N}{A_\perp} \int \frac{dx}{x} \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} \int \frac{d^2 \mathbf{q}_\perp}{(2\pi)^2} P\left(\frac{\omega}{E}\right) |R^{(1)}|^2 \langle \text{Tr} [|R^{(S_1)}|^2 + 2\text{Re} (R^{(0)\dagger} R^{(D_1)})] \rangle \\ \left[(1 + N_g(xE)) \delta(\omega - xE) \theta(1-x) + N_g(xE) \delta(\omega + xE) \right] \\ \approx \frac{\alpha_s C_2 C_R C_A}{d_A \pi} \frac{N}{A_\perp} \int \frac{dx}{x} \int \frac{d\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2} \int \frac{d^2 \mathbf{q}_\perp}{(2\pi)^2} P\left(\frac{\omega}{E}\right) (1 - v_z)^2 v^2(\mathbf{q}_\perp) \frac{\mathbf{k}_\perp \cdot \mathbf{q}_\perp}{(\mathbf{k}_\perp - \mathbf{q}_\perp)^2} \left\langle \text{Re}(1 - e^{\frac{i\omega_1 z_{10}}{1-v_z}}) \right\rangle \\ \left[(1 + N_g(xE)) \delta(\omega - xE) \theta(1-x) + N_g(xE) \delta(\omega + xE) \right], \quad (13)$$

where $v(\mathbf{q}_\perp) = 4\pi\alpha_s/(\mathbf{q}_\perp^2 + \mu^2)$.

The non-Abelian LPM effect is seen in Eq.(13) as arising from the gluon formation factor $1 - \exp(i\omega_1 z_{10}/(1 - v_z))$. The formation time of gluon radiation $\tau_f \equiv (1 - v_z)/\omega_1$ becomes shorter (longer), the LPM effect is reduced (enhanced) in the presence of collective flow in the positive (negative) jet direction, respectively. The gluon formation factor must be averaged over the longitudinal target profile, which is defined as $\langle \dots \rangle = \int dz \rho(z) \dots$. We take the target distribution as an exponential Gaussian form $\rho(z) = \exp(-z/L_e)/L_e$ with $L_e = L/2$, the gluon formation factor can be deduced as

$$\left\langle \text{Re}(1 - e^{\frac{i\omega_1 z_{10}}{1-v_z}}) \right\rangle = \frac{2}{L} \int_0^\infty dz_{10} e^{-2z_{10}/L} (1 - e^{\frac{i\omega_1 z_{10}}{1-v_z}}) \\ = \frac{(\mathbf{k}_\perp - \mathbf{q}_\perp)^4 L^2}{16x^2 E^2 (1 - v_z)^2 + (\mathbf{k}_\perp - \mathbf{q}_\perp)^4 L^2}. \quad (14)$$

After integration over \mathbf{k}_\perp and \mathbf{q}_\perp , it can be obtained that gluon formation factor is $1/(1 - v_z)$ times as the static case. So that considering the flow along the jet direction, it increases gluon formation factor and decreases LPM effect. However, square of radiation amplitude is decreased by the collective flow as can be seen from Eq. (13).

The jet energy loss can be divided into two parts. The zero-temperature part corresponds to the radiation induced by rescattering without detailed balance effect and can be expressed as

$$\Delta E_{rad}^{(1)} = \int d\omega \omega \frac{dP^{(1)}}{d\omega} \Big|_{T=0} \\ = \frac{\alpha_s C_R}{\pi} \frac{L}{l_g} E \int dx \int \frac{d\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2} \int d^2 \mathbf{q}_\perp |\bar{v}(\mathbf{q}_\perp)|^2 P(x) (1 - v_z)^2 \frac{\mathbf{k}_\perp \cdot \mathbf{q}_\perp}{(\mathbf{k}_\perp - \mathbf{q}_\perp)^2} \left\langle \text{Re}(1 - e^{\frac{i\omega_1 z_{10}}{1-v_z}}) \right\rangle, \quad (15)$$

where $l_g = C_R l / C_A$ is the mean-free path of the gluon.

The temperature-dependent part of energy loss induced by rescattering to the first order in opacity comes from thermal absorption with partial cancelation by stimulated emission, in the presence of flow it can be written as

$$\begin{aligned}\Delta E_{abs}^{(1)} &= \int d\omega \omega \left(\frac{dP^{(1)}}{d\omega} - \frac{dP^{(1)}}{d\omega} \Big|_{T=0} \right) \\ &= \frac{\alpha_s C_R L}{\pi l_g} E \int dx \int \frac{d\mathbf{k}_\perp^2}{\mathbf{k}_\perp^2} \int d^2\mathbf{q}_\perp |\bar{v}(\mathbf{q}_\perp)|^2 N_g(xE) (1-v_z)^2 \frac{\mathbf{k}_\perp \cdot \mathbf{q}_\perp}{(\mathbf{k}_\perp - \mathbf{q}_\perp)^2} \\ &\quad [P(-x) \langle Re(1 - e^{\frac{i\omega_1 z_{10}}{1-v_z}}) \rangle - P(x) \langle Re(1 - e^{\frac{i\omega_1 z_{10}}{1-v_z}}) \rangle \theta(1-x)],\end{aligned}\quad (16)$$

where $|\bar{v}(\mathbf{q}_\perp)|^2$ is defined as the normalized distribution of momentum transfer from the scattering centers,

$$|\bar{v}(\mathbf{q}_\perp)|^2 \equiv \frac{1}{\sigma_{el}} \frac{d^2\sigma_{el}}{d^2\mathbf{q}_\perp} = \frac{1}{\pi} \frac{\mu_{eff}^2}{(\mathbf{q}_\perp^2 + \mu^2)^2}, \quad (17)$$

$$\frac{1}{\mu_{eff}^2} = \frac{1}{\mu^2} - \frac{1}{q_{\perp max}^2 + \mu^2}, \quad q_{\perp max}^2 \approx 3E\mu. \quad (18)$$

To obtain a simple analytic result, we take the kinematic boundaries limit $q_{\perp max} \rightarrow \infty$, the angular integral can be carried out by partial integration. In the limit of $EL \gg 1$ and $E \gg \mu$, we obtain the approximate asymptotic behavior of the energy loss,

$$\frac{\Delta E_{rad}^{(1)}}{E} = (1-v_z) \frac{\alpha_s C_R \mu^2 L^2}{4\lambda_g E} \left[\ln \frac{2E}{\mu^2 L} - 0.048 \right] + \mathcal{O}(|\mathbf{v}|^2), \quad (19)$$

$$\frac{\Delta E_{abs}^{(1)}}{E} = -\frac{\pi\alpha_s C_R}{3} \frac{LT^2}{\lambda_g E^2} \left[\ln \frac{\mu^2 L}{T} - 1 + \gamma_E - \frac{6\zeta'(2)}{\pi^2} \right] + \mathcal{O}(|\mathbf{v}|^2). \quad (20)$$

Although the collective flow reduces the square of radiation amplitude when $v_z > 0$, it increases the gluon distribution function so that the energy gain to the first order in opacity is nearly the same as that in the static medium. The collective flow hardly change the energy gain to the first order in opacity.

Our analytic result implies, to the first order in opacity, the emitted energy loss is changed by a factor $(1-v_z)$ with collective flow. In addition, the collective flow increases the energy gain to the zeroth order in opacity with a factor $(1+v_z)^2$ and hardly change the energy gain to the first order in opacity. Our results is consistent with GLV static potential results when the velocity of the collective flow goes to zero.

Shown in Fig. 2 is the energy gain via gluon absorption with rescattering to the first order in opacity for $v_z = 0, 0.1, 0.2, 0.3$ and without rescattering as functions of E/μ . For comparison, we take the same values for the medium thickness, the mean free path, and the Debye screen mass as in Refs.[10] and [19]. The energy gain without rescattering at very small E/μ region is larger than that with rescattering if $v_z > 0.2$. This is different with the static case. But at smaller flow velocity or at higher jet energies, the energy gain without rescattering becomes smaller than that with rescattering.

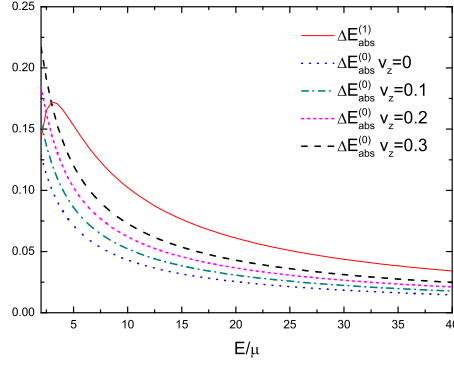


FIG. 2: The energy gain via gluon absorption with rescattering for $v_z = 0, 0.1, 0.2, 0.3$ and without rescattering as functions of E/μ .

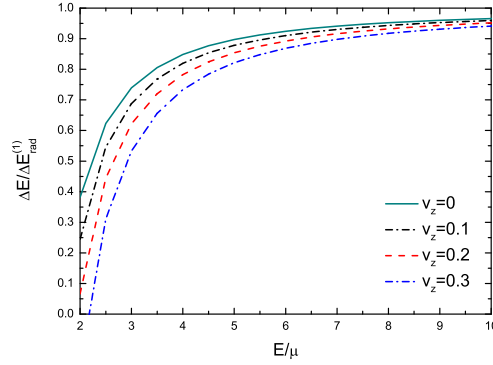


FIG. 3: The ratio of effective parton energy loss with ($\Delta E = \Delta E_{rad}^{(1)} - \Delta E_{abs}^{(1)} - \Delta E_{abs}^{(0)}$) and without ($\Delta E = \Delta E_{rad}^{(1)}$) absorption as a function of E/μ with collective flow velocity $v_z = 0, 0.1, 0.2, 0.3$ in the positive jet direction.

Shown in Fig. 3 is the ratio of the calculated radiative energy loss with ($\Delta E = \Delta E_{rad}^{(1)} - \Delta E_{abs}^{(1)} - \Delta E_{abs}^{(0)}$) and without ($\Delta E = \Delta E_{rad}^{(1)}$) thermal absorption as functions of E/μ with collective flow velocity $v_z = 0, 0.1, 0.2, 0.3$ in the positive jet direction. One sees that considering collective flow along jet direction, this ratio decreases up to 40% less than than that in the static case. Hydrodynamics tells us that flow velocity in the QGP medium is up to 0.3-0.4[20]. All these imply that for the intermediate energy jet, such as mini-jet, the gluon absorption shall be considered instead of ignoring the gluon absorption effect.

C. Second Order in Opacity Correction

It was shown by GLV [11] that in the static medium, the induced gluon radiation intensity is dominated by the first order opacity contribution, higher order correction contribute little to the radiative energy loss. Here we will analyze the second order in opacity correction with considering collective flow in the moving medium.

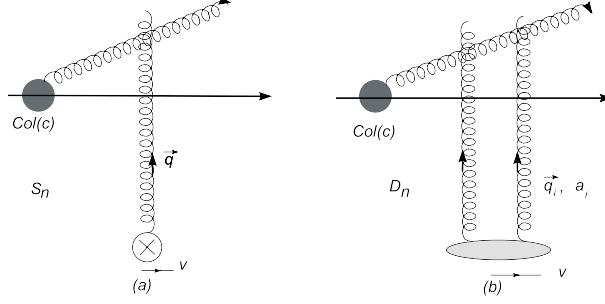


FIG. 4: Diagrammatic representation of a "direct" interaction of S_n and a double Born "virtual" interaction D_n

At the second order in opacity, consider the jet has the simplest case of two consecutive elastic rescatterings. The radiation amplitude

$$\begin{aligned}
M^{(2)} &\propto (-i)(2p - q_1 - 2q_2)_\mu A^\mu (-i)(2p - q_2)_\nu A^\nu \\
&\propto 2\pi\delta(q_1^0 - \mathbf{v} \cdot \mathbf{q}_1) e^{-i\mathbf{q}_1 \cdot \mathbf{x}_1} T_{a_1}(j) T_{a_1}(i) (2E + \mathbf{v} \cdot \mathbf{q}_1) \\
&\quad \times 2\pi\delta(q_2^0 - \mathbf{v} \cdot \mathbf{q}_2) e^{-i\mathbf{q}_2 \cdot \mathbf{x}_2} T_{a_2}(j) T_{a_2}(i) (2E + \mathbf{v} \cdot \mathbf{q}_2) R^{(2)}, \quad (21)
\end{aligned}$$

where $R^{(2)} = (1 - v_z)^2 \tilde{v}(\mathbf{q}_1) \tilde{v}(\mathbf{q}_2)$, which is changed by the collective flow with a factor $(1 - v_z)^2$.

The inclusive gluon distribution to the second order in opacity is a sum of 7^2 direct plus 2×86 virtual cut diagrams. It is useful to write the sum of amplitudes in a certain class of diagrams[11]. There are two basic iteration steps to construct the inclusive distribution of gluon. The first one represent the addition of a "direct" interaction of S_n that changes the color or momentum of the target parton with flow velocity \vec{v} located at \tilde{x}_i as illustrated in Fig. (4a). The second one corresponds to a double Born "virtual" interaction D_n that leaves both the color and momentum of the target parton unchanged as in Fig. (4b). As discussed in the last section, R^{S_1} and R^{D_1} correspond to the first order in opacity. To the second order in opacity, four new classes of factorized radiation amplitude emerge. They are readily derived from R^{S_n} and R^{D_n} with $n \leq 2$.

The first class of the factorized radiation amplitude include two single rescattering, which can be written as,

$$\begin{aligned}
R^{(S_1 S_2)} &= 2ig \cdot \epsilon_\perp (1 - v_z)^2 \left[\left(\mathbf{H} e^{\frac{i\omega_0 z_0}{1-v_z}} T_{a_2} T_{a_1} T_c + \mathbf{C}_1 e^{\frac{i(\omega_0 z_1 - \omega_1 z_{10})}{1-v_z}} T_{a_2} [T_c, T_{a_1}] \right. \right. \\
&\quad + \mathbf{C}_2 e^{\frac{i(\omega_0 z_2 - \omega_2 z_{20})}{1-v_z}} T_{a_1} [T_c, T_{a_2}] + \mathbf{B}_1 e^{\frac{i\omega_0 z_1}{1-v_z}} T_{a_2} [T_c, T_{a_1}] \\
&\quad + \mathbf{B}_2 e^{\frac{i\omega_0 z_2}{1-v_z}} [T_c, T_{a_2}] T_{a_1} + \mathbf{C}_{(12)} e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21} - \omega_{(12)} z_{10})}{1-v_z}} [[T_c, T_{a_2}], T_{a_1}] \\
&\quad \left. \left. + \mathbf{B}_{2(12)} e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21})}{1-v_z}} [[T_c, T_{a_2}], T_{a_1}] \right) \right. \\
&\quad + 2v_z \cdot \left(-\mathbf{H} e^{\frac{i\omega_0 z_0}{1-v_z}} T_{a_2} T_{a_1} T_c - \mathbf{H} e^{\frac{i\omega_0 z_1}{1-v_z}} T_{a_2} [T_c, T_{a_1}] + \mathbf{H} e^{\frac{i\omega_0 z_2}{1-v_z}} T_{a_2} T_c T_{a_1} \right. \\
&\quad \left. \left. - \mathbf{C}_{(12)} e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21})}{1-v_z}} [[T_c, T_{a_2}], T_{a_1}] + \mathbf{C}_{(12)} e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21} - \omega_{(12)} z_{10})}{1-v_z}} [[T_c, T_{a_2}], T_{a_1}] \right) \right] \quad (22)
\end{aligned}$$

where

$$\omega_{(ij\dots)} = \frac{(\mathbf{k}_\perp - \mathbf{q}_{i\perp} - \mathbf{q}_{j\perp} - \dots)^2}{2\omega} \quad , \quad \mathbf{C}_{(ij\dots)} = \frac{\mathbf{k}_\perp - \mathbf{q}_{i\perp} - \mathbf{q}_{j\perp} - \dots}{(\mathbf{k}_\perp - \mathbf{q}_{i\perp} - \mathbf{q}_{j\perp} - \dots)^2} \quad , \quad (23)$$

$$\mathbf{B}_{(i_1 i_2 \dots i_m)(j_1 j_2 \dots j_n)} = \mathbf{C}_{(i_1 i_2 \dots i_m)} - \mathbf{C}_{(j_1 j_2 \dots j_n)} \quad , \quad (24)$$

and $z_{ij} = z_i - z_j$.

Secondly, the radiation amplitude of a single hit followed by a double Born scattering can be expressed as

$$\begin{aligned} R^{(S_1 D_2)} = & 2ig \cdot \epsilon_\perp (1 - v_z)^3 \left[\left(-\frac{C_R + C_A}{2} \mathbf{H} e^{\frac{i\omega_0 z_0}{1-v_z}} T_{a_1} T_c - \frac{C_R + C_A}{2} \mathbf{C}_1 e^{\frac{i(\omega_0 z_1 - \omega_1 z_{10})}{1-v_z}} [T_c, T_{a_1}] \right. \right. \\ & - \mathbf{C}_2 e^{\frac{i(\omega_0 z_2 - \omega_2 z_{20})}{1-v_z}} T_{a_2} T_{a_1} [T_c, T_{a_2}] - \frac{C_R + C_A}{2} \mathbf{B}_1 e^{\frac{i\omega_0 z_1}{1-v_z}} [T_c, T_{a_1}] + \frac{C_A}{2} \mathbf{B}_2 e^{\frac{i\omega_0 z_2}{1-v_z}} T_c T_{a_1} \\ & - \mathbf{C}_{(12)} e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21} - \omega_{(12)} z_{10})}{1-v_z}} T_{a_2} [[T_c, T_{a_2}], T_{a_1}] \\ & \left. \left. - \mathbf{B}_{2(12)} e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21})}{1-v_z}} T_{a_2} [[T_c, T_{a_2}], T_{a_1}] \right) \right. \\ & + 2v_z \cdot \left(-\frac{C_A - C_R}{2} \mathbf{H} e^{\frac{i\omega_0 z_0}{1-v_z}} T_{a_1} T_c + \frac{C_R}{2} \mathbf{H} e^{\frac{i\omega_0 z_1}{1-v_z}} [T_c, T_{a_1}] \right. \\ & - \frac{C_R - C_A}{2} \mathbf{H} e^{\frac{i\omega_0 z_2}{1-v_z}} T_c T_{a_1} - \frac{C_A}{2} \mathbf{H} e^{\frac{i\omega_0 z_1}{1-v_z}} [T_c, T_{a_1}] + C_A \mathbf{C}_1 e^{\frac{i\omega_0 z_1}{1-v_z}} [T_c, T_{a_1}] \\ & - C_A \mathbf{C}_1 e^{\frac{i(\omega_0 z_1 - \omega_1 z_{10})}{1-v_z}} [T_c, T_{a_1}] \\ & + \mathbf{C}_{(12)} e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21})}{1-v_z}} T_{a_2} [[T_c, T_{a_2}], T_{a_1}] \\ & \left. \left. - \mathbf{C}_{(12)} e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21} - \omega_{(12)} z_{10})}{1-v_z}} T_{a_2} [[T_c, T_{a_2}], T_{a_1}] \right) \right] \quad . \quad (25) \end{aligned}$$

Interchanging the sequence of the single hit and the double Born interaction in the above process, one would find a new radiation amplitude:

$$\begin{aligned} R^{(D_1 S_2)} = & 2ig \cdot \epsilon_\perp (1 - v_z)^3 \left[\left(-\frac{C_R + C_A}{2} \mathbf{H} e^{\frac{i\omega_0 z_0}{1-v_z}} T_{a_2} T_c + \frac{C_A}{2} \mathbf{C}_1 e^{\frac{i(\omega_0 z_1 - \omega_1 z_{10})}{1-v_z}} T_{a_2} T_c \right. \right. \\ & - \frac{C_R + C_A}{2} \mathbf{C}_2 e^{\frac{i(\omega_0 z_2 - \omega_2 z_{20})}{1-v_z}} [T_c, T_{a_2}] + \frac{C_A}{2} \mathbf{B}_1 e^{\frac{i\omega_0 z_1}{1-v_z}} T_{a_2} T_c - \frac{C_R}{2} \mathbf{B}_2 e^{\frac{i\omega_0 z_2}{1-v_z}} [T_c, T_{a_2}] \\ & + \frac{C_A}{2} \mathbf{C}_{(12)} e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21} - \omega_{(12)} z_{10})}{1-v_z}} [T_c, T_{a_2}] + \frac{C_A}{2} \mathbf{B}_{2(12)} e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21})}{1-v_z}} [T_c, T_{a_2}] \left. \right) \\ & + 2v_z \cdot \left(-\frac{C_A - C_R}{2} \mathbf{H} e^{\frac{i\omega_0 z_0}{1-v_z}} T_{a_2} T_c + \frac{C_A}{2} \mathbf{H} e^{\frac{i\omega_0 z_1}{1-v_z}} T_{a_2} T_c \right. \\ & + (C_A \mathbf{C}_2 - \frac{C_A}{2} \mathbf{C}_{(12)}) e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21})}{1-v_z}} [T_c, T_{a_2}] - \frac{C_R}{2} \mathbf{H} e^{\frac{i\omega_0 z_2}{1-v_z}} T_{a_2} T_c \\ & \left. \left. - C_A \mathbf{C}_2 e^{\frac{i(\omega_0 z_2 - \omega_2 z_{20})}{1-v_z}} [T_c, T_{a_2}] + \frac{C_A}{2} \mathbf{C}_{(12)} e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21} - \omega_{(12)} z_{10})}{1-v_z}} [T_c, T_{a_2}] \right) \right] \quad , \quad (26) \end{aligned}$$

Then, if a jet parton encounters two double Born interaction, its radiation amplitude will

be

$$\begin{aligned}
R^{(D_1 D_2)} = & 2ig \cdot \epsilon_\perp (1-v_z)^4 \left[\left(\frac{(C_R + C_A)^2}{4} \mathbf{H} e^{\frac{i\omega_0 z_0}{1-v_z}} T_c - \frac{C_A(C_R + C_A)}{4} \mathbf{C}_1 e^{\frac{i(\omega_0 z_1 - \omega_1 z_{10})}{1-v_z}} T_c \right. \right. \\
& - \frac{C_A(C_R + C_A)}{4} \mathbf{C}_2 e^{\frac{i(\omega_0 z_2 - \omega_2 z_{20})}{1-v_z}} T_c - \frac{C_A(C_R + C_A)}{4} \mathbf{B}_1 e^{\frac{i\omega_0 z_1}{1-v_z}} T_c - \frac{C_A C_R}{4} \mathbf{B}_2 e^{\frac{i\omega_0 z_2}{1-v_z}} T_c \\
& + \frac{C_A^2}{4} \mathbf{C}_{(12)} e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21} - \omega_{(12)} z_{10})}{1-v_z}} T_c + \frac{C_A^2}{4} \mathbf{B}_{2(12)} e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21})}{1-v_z}} T_c \Big) \\
& + 2v_z \cdot \left(- \frac{(C_A - C_R)^2}{4} \mathbf{H} e^{\frac{i\omega_0 z_0}{1-v_z}} T_c + C_A^2 \mathbf{H} e^{\frac{i\omega_0 z_0}{1-v_z}} T_c \right. \\
& - \frac{C_A(C_R + C_A)}{4} \mathbf{H} e^{\frac{i\omega_0 z_1}{1-v_z}} T_c - \frac{C_A^2}{2} \mathbf{B}_1 e^{\frac{i\omega_0 z_1}{1-v_z}} T_c \\
& + \frac{C_A^2}{4} \mathbf{C}_{(12)} e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21} - \omega_{(12)} z_{10})}{1-v_z}} T_c - \frac{C_A^2}{2} \mathbf{C}_2 e^{\frac{i(\omega_0 z_2 - \omega_2 z_{20})}{1-v_z}} T_c \\
& - \frac{C_A^2}{2} \mathbf{C}_1 e^{\frac{i(\omega_0 z_1 - \omega_1 z_{10})}{1-v_z}} T_c + \frac{C_A^2}{4} \mathbf{B}_{2(12)} e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21})}{1-v_z}} T_c \\
& \left. \left. + \frac{C_A^2}{4} \mathbf{C}_2 e^{\frac{i(\omega_0 z_2 - \omega_2 z_{21})}{1-v_z}} T_c - \frac{C_R(C_A - C_R)}{4} \mathbf{H} e^{\frac{i\omega_0 z_2}{1-v_z}} T_c \right) \right], \tag{27}
\end{aligned}$$

We can see clearly that these amplitudes are influenced by collective flow.

We recall the three amplitudes $R^{(0)}$, $R^{(S_1)}$ and $R^{(D_1)}$ at the zeroth and first order in opacity respectively, to obtain the radiation probability to the second order in opacity,

$$\begin{aligned}
\frac{dP^{(2)}}{d\omega} = & \frac{C_R \alpha_s}{2\pi^2} \left(\frac{L}{l_g} \right)^2 \int \frac{dx}{x} \int d^2 \vec{k}_\perp \int d^2 \vec{q}_{1\perp} \int d^2 \vec{q}_{2\perp} P\left(\frac{\omega}{E}\right) J_{eff}^{(2)}(k_\perp, \mathbf{q}_\perp) \\
& \times \left[(1 + N_g(xE)) \delta(\omega - xE) \theta(1-x) + N_g(xE) \delta(\omega + xE) \right]. \tag{28}
\end{aligned}$$

Here the “emission current” to the second order in opacity $J_{eff}^{(2)}(k_\perp, \mathbf{q}_\perp)$ is,

$$\begin{aligned}
J_{eff}^{(2)}(k_\perp, \mathbf{q}_\perp) = & |R^{(2)}|^2 \left\langle Tr \left[|R^{(S_1 S_2)}|^2 + 2Re(R^{(S_1 D_2)} R^{(S_1) \dagger}) \right. \right. \\
& + 2Re(R^{(D_1 S_2)} R^{(S_2) \dagger}) + 2Re(R^{(D_1)} R^{(D_1) \dagger}) + 2Re(R^{(D_1 D_2)} R^{(0) \dagger}) \Big] \Big\rangle \tag{29} \\
\approx & (1-v_z)^8 |\bar{v}(\mathbf{q}_{1\perp})|^2 |\bar{v}(\mathbf{q}_{2\perp})|^2 \left[\left(2\mathbf{C}_1 \cdot \mathbf{B}_1 \langle Re(1 - e^{\frac{i\omega_1 z_{10}}{1-v_z}}) \rangle \right. \right. \\
& - 2\mathbf{C}_{12} \cdot \mathbf{B}_{2(12)} \langle Re(1 - e^{\frac{i\omega_{12} z_{10}}{1-v_z}}) \rangle \\
& + 2\mathbf{C}_2 \cdot \mathbf{B}_2 (\langle Re(e^{\frac{i\omega_2 z_{21}}{1-v_z}}) \rangle - \langle Re(e^{\frac{i\omega_2(z_{10} + z_{21})}{1-v_z}}) \rangle) \\
& - 2\mathbf{C}_{12} \cdot \mathbf{B}_2 (\langle Re(e^{\frac{i\omega_2 z_{21}}{1-v_z}}) \rangle - \langle Re(e^{\frac{i(\omega_{12} z_{10} + \omega_2 z_{21})}{1-v_z}}) \rangle) \Big) \\
& + v_z \left(4\mathbf{C}_1 \cdot \mathbf{B}_1 \langle Re(1 - e^{\frac{i\omega_1 z_{10}}{1-v_z}}) \rangle - 4\mathbf{C}_{12} \cdot \mathbf{B}_{2(12)} \langle Re(1 - e^{\frac{i\omega_{12} z_{10}}{1-v_z}}) \rangle \right. \\
& + (2\mathbf{C}_2 \cdot \mathbf{B}_2 - 2\mathbf{C}_2^2) (\langle Re(e^{\frac{i\omega_2 z_{21}}{1-v_z}}) \rangle - \langle Re(e^{\frac{i\omega_2(z_{10} + z_{21})}{1-v_z}}) \rangle) \\
& \left. \left. - (2\mathbf{C}_{12} \cdot \mathbf{B}_2 - 2\mathbf{C}_2 \cdot \mathbf{C}_{12}) (\langle Re(e^{\frac{i\omega_2 z_{21}}{1-v_z}}) \rangle - \langle Re(e^{\frac{i(\omega_{12} z_{10} + \omega_2 z_{21})}{1-v_z}}) \rangle) \right) \right]. \tag{30}
\end{aligned}$$

The gluon formation factors lead to non-Abelian LPM effect. To the second order in opacity, the gluon formation factors are also averaged over the target profile, which is taken as

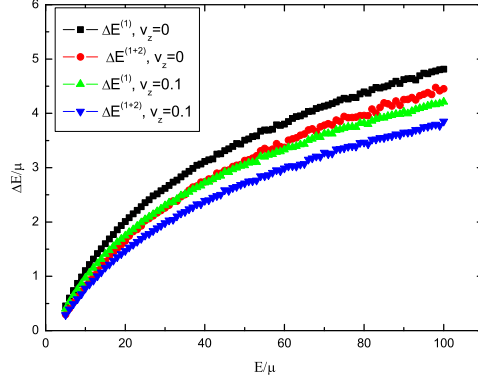


FIG. 5: The radiative jet energy loss with and without collective flow as a functions of E/μ .

$\rho(z) = \exp(-z/L_e)/L_e$ with $L_e = L/3$. This converts the gluon formation factors into simple Lorentzian factors,

$$\begin{aligned}
 \langle \text{Re}[\text{Exp}(i \frac{\sum_{j=k}^m \omega_{(k,\dots,j,\dots)} \Delta z_k}{1-v_z})] \rangle &= \int d\rho \text{Re}[\text{Exp}(i \frac{\sum_{j=k}^m \omega_{(k,\dots,j,\dots)} \Delta z_k}{1-v_z})] \\
 &= \text{Re} \prod_{j=k}^m \frac{1}{1 + i \frac{\omega_{(k,\dots,j,\dots)} L}{1-v_z} \frac{1}{3}}
 \end{aligned} \tag{31}$$

where m is the subscript of \mathbf{B} in the same term, and $k \leq j \leq m$, $0 \leq k, j, m \leq 2$. As can be seen here, to the second order in opacity, the collective flow also reduces the LPM effect if considering flow along the jet direction.

The radiative energy loss to the second order in opacity corresponds to the zero-temperature part, it can be expressed as,

$$\begin{aligned}
 \Delta E_{rad}^{(2)} &= \int d\omega \omega \frac{dP^{(2)}}{d\omega} \Big|_{T=0} \\
 &= \frac{C_R \alpha_s}{2\pi^2} \left(\frac{L}{l_g} \right)^2 E \int dx \int d^2 \vec{k}_\perp \int d^2 \vec{q}_{1\perp} \int d^2 \vec{q}_{2\perp} P\left(\frac{\omega}{E}\right) J_{eff}^{(2)}(k_\perp, \mathbf{q}_\perp),
 \end{aligned} \tag{32}$$

The numerical result of radiative energy loss as a function of E/μ is shown in Fig.(5). We take the same values for the medium thickness, the mean free path, and the Debye screen mass as in Refs.[10] and [19] for comparison. It shows that in the presence of collective flow, the second order in opacity correction is relatively small as compared to the first order in opacity. The contribution of the first order is dominant.

In the moving medium, to the second order in opacity, the energy gain via gluon absorption corresponding to the temperature-dependent part of energy loss also comes from the

partial cancelation by stimulated emission, it can be written as,

$$\begin{aligned}
\Delta E_{abs}^{(2)} &= \int d\omega \omega \left(\frac{dP^{(2)}}{d\omega} - \frac{dP^{(2)}}{d\omega} \Big|_{T=0} \right) \\
&= \frac{C_R \alpha_s}{2\pi^2} \left(\frac{L}{l_g} \right)^2 E \int dx \int d^2 \vec{k}_\perp \int d^2 \vec{q}_{1\perp} \int d^2 \vec{q}_{2\perp} J_{eff}^{(2)}(k_\perp, \mathbf{q}_\perp) \\
&\quad \times N_g(xE) [P(-x) - P(x)\theta(1-x)].
\end{aligned} \tag{33}$$

We calculated the numerical result and find that the energy gain to the second order in opacity is little as compared to the energy gain to the first order in opacity. So that it is negligible to the total effective energy loss.

IV. CONCLUSION

In summary, we have derived a new potential for the interaction of a hard jet with the parton target. It can be used to study the jet quenching phenomena in the presence of collective flow of the quark-gluon medium. With this new potential, we have investigated the effect of collective flow on jet energy loss with detailed balance. Collective flow along the jet direction leads to decreased LPM effect and the square of radiation amplitude. To the zeroth order in opacity, the energy gain without rescattering is $(1 + v_z)^2$ times as in the static medium. To the first order in opacity, the gluon emission energy loss is $(1 - v_z)$ times as that in the static medium, but the energy gain is nearly the same as that in the static medium. To the second order in opacity, both the radiative energy loss and the energy gain are relatively small as compared to the first order. All these imply the collective flow along the jet direction decreases gluon emission energy loss, increases energy gain, so that the total effective energy loss is decreased by collective flow. All these lead to that for the intermediate energy jet, such as mini-jet, the gluon absorption shall be considered. Compared to calculations for a static medium, our results will affect the suppression of high p_T hadron spectrum and anisotropy parameter v_2 in high-energy heavy-ion collisions. Our new potential can also be used for heavy quark energy loss calculation and will alter the dead cone effect of heavy quark jets. Our results shall have implications for comparisons between theory and experiment in the future.

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